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Evaporative cooling of water in a natural draft cooling tower

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Abstract

A mathematical model of the performance of a cooling tower is presented. The model consists of two interdependent boundary-value problems, a total of 9 ODE, and the algorithm of self-consistent solution. The first boundary-value problem describes evaporative cooling of water drops in the spray zone of a cooling tower; the second boundary-value problem describes film cooling in the pack. Simulation of the boundary-value problems has been made. The comparison between experimental data and calculated results showed that the model correctly describes the basic regularities of the cooling tower performance. In the effective droplet-radius approximation the difference in the thermal efficiency between calculated and experimental results does not exceed 3%. The limits of applicability of the developed mathematical model and its possibilities are discussed.

1. Introduction

Natural draft cooling towers are intended for cooling circulating water in thermal and atomic power plants and at industrial enterprises [1,2]. Typical modern cooling towers have the following parameters: tower height $H_t \approx 100$ m, basement diameter $D_b \approx 70$ m, and water flow rate 30,000 tons/h or more. A better understanding of evaporative cooling of water in cooling towers is necessary both for modernization of the existing cooling towers, and for predicting the efficiency of newly designed ones. Because the development of new types of cooling towers requires the conduction of many thermal and hydraulic tests that are too costly [3], the creation of a mathematical model to study the influence of a number of parameters on cooling tower performance is a very urgent problem.

In a natural draft cooling tower, water is cooled in both film flow down the sheets of a pack and in flow of falling variously sized droplets (Fig. 1). There is interference between the heat and mass transfer processes in the spray zone and in the sheets of the pack. Because the air is heated and saturated by water as it rises through the film flow zone, the effectiveness of evaporative cooling of water droplets in the spray zone decreases. On the other hand, additional heating of air by the heat transferred from the droplets in the spray zone increases the velocity of convective air flow in the cooling tower and, as a consequence, increases the intensity of evaporative cooling in the film flows. The contribution of heat and mass transfer in the rain zone of a cooling tower can also be included into the mathematical model in the same approximation but with another radius. Observations show that the droplets and jets in the rain zone of a cooling tower are formed on shedding of water from the sheets of the pack. As a rule, the radius of the droplets is quite large, and an appreciable fraction of water falls dawn in the form of jets. As this takes place, the mean radius of droplets in the rain zone may several times exceed the radius of droplets in the spray zone. Therefore, in what follows we shall neglect the evaporative cooling of water in the rain zone.

Our mathematical model of evaporative cooling of water under steady-state operational conditions of a cooling tower makes it possible to calculate the joint evaporative cooling of water droplets and films, i.e. to determine the thermal efficiency of the cooling tower as a whole. Our calculations show that the circulating water in a cooling tower is mainly cooled on the sheets

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Nomenclature

a thermal diffusivity ($m^2\,s^{-1}$)
c specific heat ($J\,kg^{-1}\,K^{-1}$)
C coefficient of aerodynamic drag, dimensionless
d space between two sheets (m)
D diffusion coefficient for water vapor (m^2/s)
g free fall acceleration (m/s^2)
h film thickness (m)
H height (m)
k coefficient defined in Eq. (3)
m droplet mass (kg)
N number of droplets in unit volume (m^3)
q specific mass flow rate per unit sheet length (kg/m s)
Q specific mass flow rate ($kg/m^2\,s$)
P dimensionless parameter
r latent heat of vaporization ($kJ\,kg^{-1}$)
R radius (m)
S dimensionless parameter
T temperature, °C
v velocity in z direction (m/s)
y coordinate in horizontal direction
z coordinate in vertical direction
Nu Nusselt number
Pr Prandtl number
Sh Sherwood number
Re Reynolds number

Greek symbols
α heat transfer coefficient, ($W\,m^{-2}\,K^{-1}$)
γ mass transfer coefficient ($kg\,m^{-2}\,s^{-1}$)
η thermal efficiency of a cooling tower, dimensionless
ρ density (kg/m^3)
ψ relative humidity of air, dimensionless
λ thermal conductivity of air, (W/m °C)
σ surface tension of water ($kg\,m^{-1}$)
μ dynamic viscosity of air ($kg\,m^{-1}\,s^{-1}$)
ν kinematic viscosity (m^2/s)
Δ difference
Σ residual, dimensionless

Subscripts
0 initial
a air
b tower base
d for drop
f for film
h horizontal
m mixture
s saturated
t for tower
v vapor
w water
eff effective
lim limiting
min minimum
max maximum
out out
th tower throat

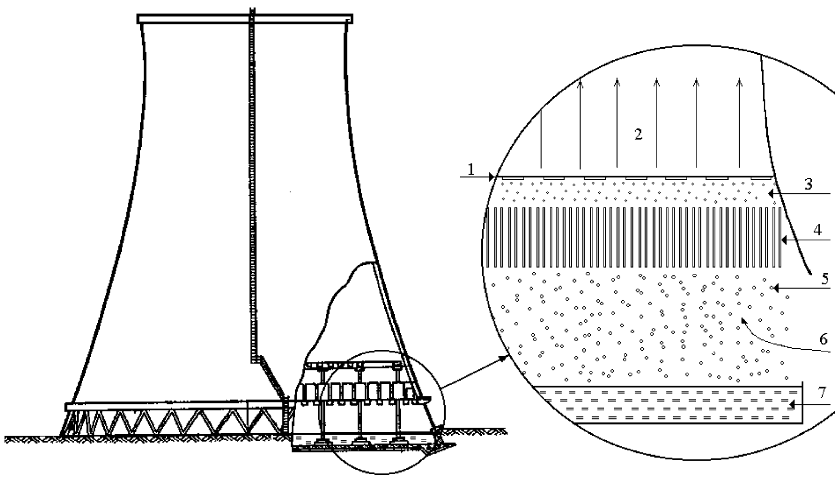

Fig. 1. Scheme of the zones of heat and mass transfer in a cooling tower. (1) Water entering the cooling tower; (2) moist air; (3) droplets in the spray zone; (4) sheets of the pack; (5) droplets in rain zone; (6) surrounding air; (7) water-collecting pond.

of the pack in film flows. Interaction between various heat and mass transfer zones in the cooling tower is carried out via moist air and hot water. Our model makes it possible to predict such important parameters as the flow rate of evaporated water, temperature, humidity and the upward velocity of the air flow. The detailed description which we propose for the hydrodynamic and heat and mass transfer processes makes it possible to find the distribution of these parameters in the analyzed zones of a cooling tower.

The basic aspects of the physical kinetics of heat- and mass-transfer processes in evaporation are outlined in [4]. At present, there are relatively few publications on numerical modeling of the processes of evaporative cooling of water in cooling towers. For example, in [3] a numerical model of thermal performance of a splash pack in a counter-flow cooling tower is developed. In [5] our preliminary simulations of the coupled cooling effects of droplets and films in a cooling tower are presented. In [6] three nonlinear differential equations are derived from the energy and mass conservation laws for finding the enthalpy and temperature of moist air leaving a cooling tower. Unlike our model, that model assumes the initial and final temperature of the water, the wet bulb temperature and the mass flow rates of water and air to be given. Evaporative cooling of water in film flows on an adiabatic wall is the concern of [7–9]. Evaporative cooling of falling water droplets is considered in [10–12]. As a rule, in all of the investigations that were carried out complex jet-droplet flows in a cooling tower were replaced by an ensemble of equally sized droplets. We also employ this approximation. The size of a droplet in an ensemble is equal to some effective size that insures the known efficiency of complex jet-droplet water flow cooling at a given water flow rate, initial water temperature, temperature and humidity of an air-steam mixture. The procedure of determining this effective size is discussed below.

We describe the efficiency of evaporative cooling of water by means of the thermal efficiency of a cooling tower:

$$\eta = \frac{T_{w0} - T_{w_{out}}}{T_{w0} - T_{lim}}, \tag{1}$$

where T_{w0} is the temperature of hot water entering the cooling tower, $T_{w_{out}}$ is the temperature of water leaving the cooling tower, T_{lim} is the limiting temperature for evaporative cooling of water, which is equal to the wet-bulb temperature. T_{lim} can be obtained from the condition [2]

$$\rho_s(T_{lim}) = \psi\rho_s(T_a), \tag{2}$$

where ρ_s is the density of saturated vapor, ψ is the relative humidity of air, T_a is the temperature of the surrounding air.

It follows from dimensional analysis that the dimensionless quantity η should depend only on dimensionless parameters. Our investigation showed that the most essential of these parameters is the ratio of the specific mass flow rates of water and air Q_w/Q_a [13]. The specific flow rate of air Q_a depends on the height of a tower and on the difference between the air density inside and outside the cooling tower, which depends on the temperature and flow rate of water.

The preliminary simulations of the cooling tower performance based on our mathematical model of film cooling, just as the comparison of the calculated results with experimental data were presented in [5,14]. We should also note that the efficiency of evaporative heat transfer in a cooling tower is influenced by the wind that may rise near the cooling tower [15]. Our model is intended for investigations under calm and weak wind conditions. The cooling tower aerodynamics and, in particular, the inlet aerodynamics also influence the thermal efficiency of a tower [16]. Therefore, we shall begin with a brief description of the aerodynamics necessary for calculating the cooling tower performance.

2. Elements of the cooling tower aerodynamics

The upward moist air velocity v_a between the sheets of the pack of the cooling tower is calculated by means of the following expression:

$$v_a = (R_{th}/R_b)^2((2gH_t\Delta\rho_m)/\rho_m)^{0.5}k, \tag{3}$$

where R_{th} are the radius of the throat and R_b of the basement of the cooling tower respectively; H_t is the height from the upper edge of the pack to the cooling tower throat. This expression is obtained from the standard expression for the convective flow velocity in a cooling tower and from the continuity equation for the moist air flow through the pack and the cooling tower throat. In the numerical calculations presented in the paper, the empirical coefficient k is equal to 0.5. This value was found as a result of the treatment of our experimental data [17]. The change in the moist air density $\Delta\rho_m$ is calculated in the process of self-consistent solution of the system of the differential equations that describe the processes of heat and mass transfer in the cooling tower. The value of $\Delta\rho_m$ is determined in the main by the change of the moist air temperature in the pack. The moist air velocity in the zone of heat and mass transfer is taken to be constant, because the size of this zone is much less than the cooling tower height. It should be noted that our mathematical model makes it possible to determine the coefficient k for any arbitrary cooling tower provided the corresponding data of thermal measurements are used.

Expression (3) describes the internal aerodynamics of the cooling tower in one-dimensional approximation. More accurate simulation of evaporative cooling requires considerations of the effects of non uniformity in the distribution of air flows in a cooling tower, and our studies in this direction are now in progress. At the present time the authors are not aware about the experimental data on the internal aerodynamics of a cooling tower.

3. Simulation of evaporative cooling of water in cooling towers

Eq. (3) allows one to carry out calculations of heat and mass transfer processes in evaporative cooling of water. Below, we describe heat and mass transfer processes in the approximation of averaged temperatures and averaged water vapor density.

3.1. Simulation of evaporative cooling of water in film flows

As already mentioned above, for small and medium hydraulic loadings of a cooling tower water is mainly cooled in film flows. Film flows in a cooling tower are formed during water drain down the sheets of the pack. It is obvious that the efficiency of evaporative cooling of water films in a cooling tower depends on many parameters. The most important of them are the following: the water mass flow rate Q_w, the initial temperature T_{f0} of a water film, the initial temperature T_{a0} and relative humidity ψ of air at the inlet to the pack, and also the velocity of air v_a between the sheets of the pack. Of great importance are the following geometric parameters: the height of the sheets in the pack H_f and the distance d between two neighboring sheets.

We consider evaporative cooling of thin layers of water of thickness h flowing down two vertical sheets. Fig. 2 displays the diagram of the problem under investigation. Evaporative cooling of a film flowing between two vertical sheets represents "an elementary cell" of a cooling tower. As the air moves upward, it is heated and the concentration of water vapor increases in it, thus slowing down the intensity of heat and mass transfer with descending water films. For evaporative cooling in towers the condition $H_f \gg d \gg h$ is valid, which allows one to substantially simplify the mathematical formulation of the problem.

After averaging the transverse profiles of the water film temperature, air velocity, moist air temperature, and vapor density in air, the system of the equations that describe the processes of heat and mass transfer between the vertically descending water film and the ascending moist air includes the following equations:

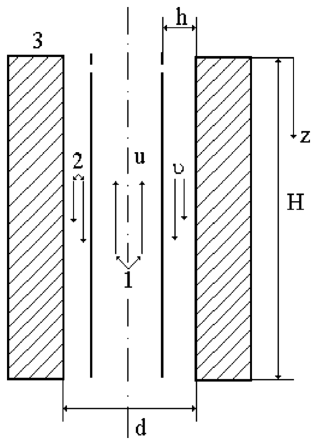

Fig. 2. Schematic of the air and water film flows on vertical sheets. (1) Air flow; (2) water film; (3) adiabatic sheet.

The equation describing the change in the film thickness $h(z)$ due to evaporation

$$\frac{dh(z)}{dz} = -\frac{\gamma_f(Re)(\rho_s(T_f(z)) - \rho(z))}{\rho_w v_f}; \tag{4}$$

the equation determining the change in the cross-section averaged water film temperature $T_f(z)$ due to the contact with the cold air and due to evaporation

$$\frac{dT_f(z)}{dz} = \frac{\alpha_f(Re)(T_a(z) - T_f(z)) - r\gamma_f(Re)(\rho_s(T_f(z)) - \rho(z))}{c_w \rho_w h(z) v_f}; \tag{5}$$

the equation for calculating the change in the cross-section averaged temperature of the moist air $T_a(z)$

$$\frac{dT_a(z)}{dz} = -\frac{2\alpha_f(Re)(T_f(z) - T_a(z))}{v_a d_1(\rho_a c_a + \rho c)}; \tag{6}$$

the equation describing the change in the density of the water vapor $\rho(z)$ in air

$$\frac{d\rho(z)}{dz} = -\frac{2\gamma_f(Re)(\rho_s(T_f(z)) - \rho(z))}{v_a d_1}. \tag{7}$$

We note that $d_1 = d - 2h_f$ where h_f is the film thickness that depends on a hydraulic load.

The system of differential equations (4)–(7) was integrated numerically with the following boundary conditions:

at $z = 0$

$$h|_{z=0} = h_0; \tag{8}$$

$$T_f|_{z=0} = T_{f0}; \tag{9}$$

at $z = H_f$

$$T_a|_{z=H_f} = T_{a0}; \tag{10}$$

$$\rho|_{z=H_f} = \rho_s(T_{a0})\psi = \rho_{v0}. \quad (11)$$

In Eqs. (5) and (6), $\alpha(Re)$ is the heat-transfer coefficient determined as follows [18]:

$$\alpha_f = \frac{0.324 Re^{0.5} Pr^{0.33} \lambda_a}{x}, \quad (12)$$

where Pr is the Prandtl number.

For our problem the Reynolds number is determined [18,19] as

$$Re = \frac{x\rho_a(v_a + v_f)}{\mu_a}, \quad (13)$$

where in (12) and (13) λ_a is the thermal conductivity of air; x is determined as $x = H_f - z$; $v_a + v_f$ is the relative velocity of the air flow past the water film; μ_a is the dynamic viscosity of air. It is important to note that a rigorous description of our problem would require the use of three Reynolds numbers Re: for the film flow, for the vapor–air mixture flow between the sheets, and for the processes of interphase heat and mass exchange. We use only the Reynolds number (13), which remains after averaging all the corresponding parameters over the film and vapor–air mixture.

Using the analogy between the heat transfer and mass transfer processes, since the Nusselt number ($Nu = \alpha(H_f - z)/\lambda$) and the Sherwood number ($Sh = \gamma(H_f - z)/D$) have the same dependence on the Reynolds number, we can determine the mass-transfer coefficient γ_f in Eqs. (4), (5) and (7). Then for the coefficient of mass transfer between the laminar air flow and a thin film of water we have the following expression:

$$\gamma_f = \frac{0.324 Re^{0.5} Pr^{0.33} D}{x}. \quad (14)$$

In the calculations, we took into account the temperature dependence of the diffusion coefficient of water vapor D [20].

In Eqs. (4) and (5) the velocity of the descending water film, averaged over the cross section, was determined in a laminar approximation [19,21] as

$$v_f = \left(\frac{g}{3\nu_w}\right)^{\frac{1}{3}} \left(\frac{Q_w}{\rho_w}\right)^{\frac{2}{3}}. \quad (15)$$

The boundary-value problem (4)–(11) for the system of nonlinear ordinary differential equations (4)–(7) was solved by a "shooting" method [22]. The numerical solution of the system of the differential equations was obtained by the Runge-Kutta method of fourth order. The accuracy was controlled by means of the residual criterion Σ_f:

$$\Sigma_f = \sqrt{\left(\frac{T_a(H_f) - T_{a0}}{T_{a0}}\right)^2 + \left(\frac{\rho_v(H_f) - \rho_{v0}}{\rho_{v0}}\right)^2} \quad (16)$$

The solution was terminated as soon as the condition $\Sigma_f < 10^{-4}$ was satisfied. In fact, a further increase in the accuracy of calculations did not influence the water temperature and other parameters.

The calculation by the film model have shown that increase in height of the sheets by more than 3 m practically does not affect the efficiency of evaporative cooling because of the effect of saturation of moist air. Moreover, it is known that the increase in the length of the sheets results in formation of water jets [23] because of the instability of the film flow. In addition to the length of the sheets, an important parameter is also the distance d between two neighbouring sheets. As d increases, the air flow rate between the sheets increases. As a rule, the distance between the sheets of the pack in a cooling tower is about $d \approx 2.5$ cm, since a further increase in d increases the specific water flow rate per unit length of a sheet with further obvious negative consequences. The film cooling efficiency η_f is shown in Fig. 3. It was calculated by means of our mathematical model, in which the dimensionless parameter P includes the distance between the sheets of a pack d, the velocity of the ascending air flow v_a, the diffusion coefficient of water vapor D and the height of the sheets in the pack H_f:

$$P = \frac{d^2 v_a}{D H_f}.$$

As is seen in Fig. 3, η_f does not practically increase when $P \geqslant 10$.

The qualitative analysis of the mathematical model of evaporative cooling of water was carried out in [7]. It

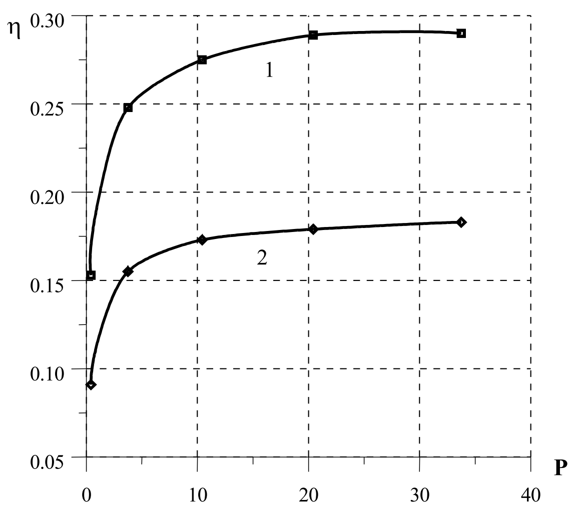

Fig. 3. Thermal efficiency of film cooling η_f versus the dimensionless parameter P. Curve 1 is the specific water flow rate per unit sheet length $q_L = 0.021$ kg/m s; curve 2 is for $q_L = 0.038$ kg/m s. $T_{f0} = 40$ °C, $T_{a0} = 20$ °C, $\psi = 70\%$, $H_f = 3$ m.

gave the possibility of obtaining the approximate formulas that relate the geometrical dimensions, thermophysical properties and the flow rates values of the phases. In particular, it is shown that the thermal efficiency η_f depends on many parameters in accordance with the following expression:

$$\eta_f \sim \frac{Q_a}{Q_w}\frac{H_f}{d}\frac{\left(1+\frac{v_f}{v_a}\right)}{\mu_a c_w Re^{0.5}}\left[\lambda_a \frac{T_{f0}-T_{m0}}{T_{f0}-T_{\lim}} + Dr\frac{\partial \rho_s}{\partial T}\bigg|_{T=T_{\lim}}\right]. \tag{17}$$

The specific flow rate of air per unit length of the sheets is equal to $d_1\rho_a v_a$. It follows from Eq. (17) that η_f depends on many parameters of a cooling tower; its value is influenced by both the evaporation of water and heat transfer. We emphasize that η_f is inversely proportional to Q_w/Q_a.

Some of the results of numerical integration of the system (4)–(11) are represented in Fig. 4. From the plots given in Fig. 4 it is seen that the process of evaporative cooling is most intense in the lower part of the sheets of the pack. From the physical point of view, this effect is due to the smaller thickness of the boundary layer of the moist air above the water film and the greater difference between the thermodynamic parameters of two interacting phases.

It was mentioned above that the cooling of the water film depends substantially on the water mass flow rate

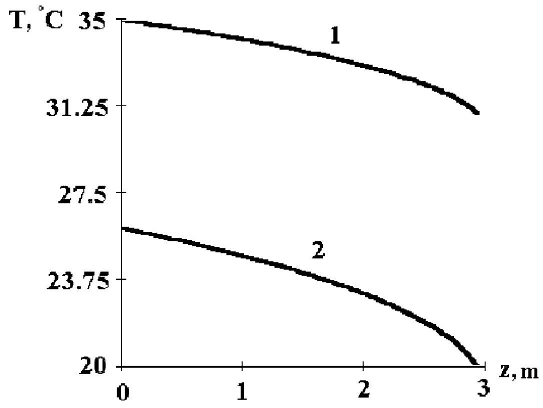

Fig. 4. Temperature profiles of water and air for film cooling. Curve 1 is the water temperature; curve 2 is the moist air temperature. $T_{f0} = 35$ °C, $T_{a0} = 20$ °C, $\Psi = 0.7$, $q_w = 0.025$ kg/m s, $H_f = 3$ m.

q_w per unit length of the sheet. If q_w increases, the thickness and the mean velocity of the water film increase. As a result, the residence time of an element of the film decreases, and the efficiency of film cooling is reduced too. Our numerical experiments show that the thermal efficiency of the pack η_f is nonlinear, monotonically decreasing function of the relation of the specific mass flow rates of the water and the air, Q_w/Q_a. The calculated values of η_f agrees well with the values η, found by treatment of the results of full-scale measurements [17].

3.2. Simulation of evaporative cooling of water droplets

Droplets in the wet cooling tower are formed during water sputtering by sprinklers and during water runoff from the sheets of the pack to the pond (Fig. 1). In a cooling tower, the contribution of the cooling of droplets to the heat balance of the tower depends mainly on their radius. The radius of the droplets in the spray zone of the cooling tower depends on the water flow rate: the higher the water flow rate, the smaller is the droplet size due to the larger pressure drop on sprinklers. Our calculations show that the dependence of the radius of the droplets in the spray zone on the hydraulic load is attributable to the design of the sprinkler nozzle and is not associated with the phenomenon of breaking of the droplets. Even at a maximum hydraulic load, the velocity of the droplets leaving the sprinkler is not sufficient for breaking.

The maximum radius of the droplet falling with the velocity v_d is determined from the equality of the contributions of the aerodynamic drag force and the surface tension. Droplets are not broken provided the following equation is valid [19]:

$$R_d \leqslant 2.3\frac{\sigma}{\rho_a v_d^2}, \tag{18}$$

where σ is the coefficient of the surface tension of water. This coefficient depends on temperature.

In the spray zone the minimal size of the droplets participating in the process of evaporative cooling is determined by the air ascending flow velocity v_a. If the force of aerodynamic resistance exceeds that of the gravity which is true for rather small droplets, the droplets are carried away by the ascending air flow. The

Table 1
Dependence of the minimal droplet radius $R_{d\min}$ on the convection velocity v_a

Convection velocity v_a, m/s	0.5	1	1.5	2	2.5
$R_{d\min}$, m	5.8×10^{-5}	1×10^{-4}	1.5×10^{-4}	2.0×10^{-4}	2.5×10^{-4}

dependence of the minimal size of droplets on the convection flow velocity is given in Table 1.

Let us now calculate the evaporative cooling of water droplets. We direct the z-axis vertically downward and fix the coordinate origin at the point of the beginning of droplet fall. Then the velocity of the falling water droplet $v_d(z)$ will be positive and the velocity of the ascending moist air v_a will be negative. The falling droplet experience the action of the gravity force and the force of aerodynamic drag. As the air moves upward, it is heated and its humidity increases, while the temperature of the falling droplet decreases. The system of the differential equations used to calculate the processes of heat and mass transfer between the falling droplet and the ascending moist air includes the following equations:

The equation describing the change in the droplet radius $R_d(z)$ due to evaporation

$$\frac{dR_d(z)}{dz} = -\frac{\gamma_d(Re_d)[\rho_s(T_d(z)) - \rho_v(z)]}{\rho_w v_d(z)}, \tag{19}$$

the equation determining the change in the velocity $v_d(z)$ of the falling droplet

$$\frac{dv_d(z)}{dz} = \frac{g}{v_d(z)} - C(Re_d) \cdot \frac{\rho_a[v_d(z) - v_a]^2}{2v_d(z)} \frac{\pi R_d(z)^2}{m}, \tag{20}$$

the equation for calculating the volume-averaged temperature of the droplet $T_d(z)$

$$\frac{dT_d(z)}{dz} = \frac{3\{\alpha_d(Re_d)[T_a(z) - T_d(z)] + \gamma_d(Re_d)r[\rho_s(T_d(z)) - \rho_v(z)]\}}{c_w \rho_w R_d(z) v_d(z)}, \tag{21}$$

the equation for calculating the averaged temperature of the moist air $T_a(z)$

$$\frac{dT_a(z)}{dz} = \frac{4\pi R_d(z)^2 N_d}{\rho_a c_a(v_d(z) - v_a)}[\alpha_d(Re_d)[T_a(z) - T_d(z)]], \tag{22}$$

the equation for describing the change in the density of the water vapor $\rho(z)$ in moist air

$$\frac{d\rho_v(z)}{dz} = -\frac{4\pi R_d(z)^2 N_d}{v_d(z) - v_a}\gamma_d(Re_d)[\rho_s(T_{wd}(z)) - \rho_v(z)]. \tag{23}$$

For droplets in the spray zone, described by the system of Eqs. (19)–(23), the boundary conditions are written as follows. At $z = 0$ (point of beginning of droplet fall) the initial values are defined for: the droplet radius

$$R_d|_{z=0} = R_{d0}, \tag{24}$$

the droplet temperature

$$T_d|_{z=0} = T_{d0}, \tag{25}$$

the droplet velocity

$$v_d|_{z=0} = 0. \tag{26}$$

At $z = H_d$ (the point at which air leaves the sheets of the pack, the final point of the fall of the droplet) assigned are: the air temperature at the outlet from the pack

$$T_a|_{z=H_d} = T_{a0}, \tag{27}$$

the density of the water vapor in the air

$$\rho_v|_{z=H} = \rho_{v0}. \tag{28}$$

Thus, the system of ordinary differential equations (19)–(23) and boundary conditions (24)–(28) represent the nonlinear boundary-value problem. It is important to emphasize that the influence of the number of droplets N_d on the parameters of moist air are taken into account in our model. The number of droplets per unit volume, N_d, is defined by the water flow rate and the droplet size. The specific water flow rate Q_w and density of water droplets in unit volume N_d are related by the equation

$$N_d = \frac{3Q_w}{4\rho_w \pi R_d^3 v_d}. \tag{29}$$

In accordance with [18], the coefficient of heat transfer of the droplets in the air medium, α_d, in Eqs. (21) and (22) was determined from the following dimensionless relation:

$$Nu_d = 2 + 0.5Re_d^{0.5}. \tag{30}$$

For droplets the Nusselt number is $Nu_d = 2R_d\alpha_d(Re)/\lambda_a$, and the Reynolds number is defined as

$$Re_d = \frac{2\rho_a R_d[(v_d - v_a)^2 + v_{dh}^2]^{0.5}}{\mu_a}.$$

Using the analogy between the heat transfer and mass transfer processes, for a droplet falling in an ascending air flow the mass transfer coefficient γ_d was determined as

$$\gamma_d = \frac{D(2 + 0.5Re_d^{0.5})}{2R_d(z)}. \tag{31}$$

For aerodynamic drag force of a droplet, the function $C(Re_d)$ was calculated by means of the formula [19]

$$C(Re_d) = \frac{24}{Re_d}\left(1 + \frac{1}{6}Re_d^{2/3}\right). \tag{32}$$

In our calculations the additional increase in the elevation of a droplet with growth of a hydraulic load is taken into account as well as the horizontal component velocity of falling droplets along with the vertical component of the velocity. The horizontal component of the droplet velocity influences the heat and mass transfer coefficients via the Reynolds number. This effect is not taken into account in simulation of an evaporative cooling of droplets in [5]. The value of the horizontal component of the velocity is determined by the design of the cooling tower sprinkler.

Before passing to numerical treatment of the system of Eqs. (19)–(28), we qualitatively analyze this mathematical model [24]. Qualitative estimations are obtained by approximate analytical integration of the system of equations. For the steady-state velocity of a falling droplet of radius R_{d}, the qualitative estimations were made in [10]. For the change in the droplet temperature ΔT_{d} in the spray zone of the cooling tower, for the accelerated motion we have

$$\Delta T_{\mathrm{d}} \sim \{\lambda[T_{\mathrm{a0}} - T_{\mathrm{d0}}] + Dr[\rho_{\mathrm{s}}(T_{\mathrm{d0}}) - \rho_{v0}]\} \times H_{\mathrm{d}}^{0.5}\sqrt{v_{\mathrm{a}}^2 + v_{\mathrm{dh}}^2}R_{\mathrm{d0}}^{-3/2}. \quad (33)$$

This estimate of ΔT_{d} is the lower estimate of the evaporative cooling of a droplet over the path H. Note should be made of the strong inverse dependence of ΔT_{d} on the droplet radius. The boundary-value problem of evaporative cooling of droplets was solved also by the "shooting method" [22]. To obtain numerical solution of the system of differential equations, the Runge–Kutta method of the fourth order was used. The accuracy was checked by means of the residual criterion Σ_{d}:

$$\Sigma_{\mathrm{d}}(T_{\mathrm{a0}}, \rho_{\mathrm{v0}}) = \sqrt{\left(\frac{T_{\mathrm{a}}(H_{\mathrm{d}}) - T_{\mathrm{a0}}}{T_{\mathrm{a0}}}\right)^2 + \left(\frac{\rho_{\mathrm{v}}(H_{\mathrm{d}}) - \rho_{\mathrm{v0}}}{\rho_{\mathrm{v0}}}\right)^2}, \quad (34)$$

where $T_{\mathrm{a}}(H_{\mathrm{d}})$ and $\rho_{\mathrm{v}}(H_{\mathrm{d}})$ are the results of calculation of the boundary-value problem (19)–(28) for the temperature of the air and vapor density at the point of its entrance. The solution of the problem terminated as soon as the condition $\Sigma_{\mathrm{d}} < 10^{-4}$ was fulfilled.

Experimental investigations of the thermal efficiency of evaporative cooling of falling water drops were published in [12] for the conditions similar to the conditions of cooling tower performance. Our mathematical model describes these experimental data with a high accuracy.

3.3. Simulation of joint evaporative cooling of water droplets and films in a cooling tower

For correct calculation of the cooling of water droplets in the spray zone of a cooling tower, it is necessary to take into account the increase in the temperature of moist air and the increase in the humidity of air in comparison with the inlet parameters. In turn, for correct calculation of film cooling in a cooling tower, it is necessary to take into account the change in the velocity of the ascending flow of air due to heat and mass transfer on droplets in the spray zone, plus the decrease in the water temperature in comparison with the initial temperature of water T_{w0}. Moreover, with increase in the temperature and humidity of air the convective flow velocity of air v_{a} increases (see expression (3)). The increase in the velocity of the convective flow intensifies the cooling of water droplets and films.

To carry out simulation of the steady-state process of evaporative cooling of water droplets and films in a cooling tower, an iterative algorithm has been developed based on the mathematical model of film evaporative cooling (4)–(11) and the mathematical model of evaporative cooling of droplets (19)–(28). For the given water flow rate q_{w} and the initial water temperature T_{w0}, the water film cooling is calculated without account for the contribution of heat and mass transfer on droplets. As result of calculation, the film model yields the ascending velocity of air v_{a}, the air temperature T_{a} and the density of water vapor in the moist air leaving the pack. Thereafter these data are used to calculate the evaporative cooling of water droplets in the spray zone. With account for the additional heating of the air during the evaporative cooling of droplets, the new velocity of air convection was determined. For this velocity, the evaporative cooling of water films was calculated again. The final temperature of water droplets in the spray zone was the initial one for the films. The results of calculation of the film flow again gave new values of the velocity, temperature and the density of water vapor in the flow going out of the pack of the cooling tower. After this, the evaporative cooling of droplets was calculated again. The procedure was repeated several times until the thermal efficiency of the cooling tower stopped changing as a result of iterations. Usually, 3–4 iterations are required to find a self-consistent solution.

The above-described algorithm of joint solution of two boundary-value problems enables one to calculate the outlet temperature of water in a cooling tower, the density of water vapor in moist air and its temperature. The temperature of water droplets in a cooling tower versus the effective droplet radius is shown in Fig. 5. The intersection of curve 1 with point 2 yields the effective radius for these conditions, i.e., the radius of droplets in the spray zone ensuring the coincidence of the calculated and experimental values of the thermal efficiency.

Let us consider some results of application of an iterative algorithm to the simulation of the cooling tower performance. In Fig. 6, the calculated thermal efficiency of a cooling tower and of the film cooling is given in relation to the ratio of the mass flow rates of the water and air. The air flow rate was calculated on the basis of Eq. (3) and our mathematical model of evaporative cooling. We note that the flow rare of water is the same for all the cases presented. From the plots of Fig. 6 it is seen, that with increase in the temperature of entering water (points A and A_1) the efficiency of a cooling tower

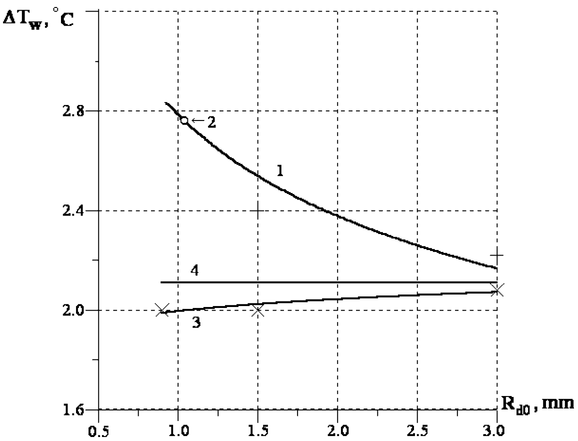

Fig. 5. Water temperature drop ΔT_w versus the water droplet radius R_0. Curve 1 is the calculated value; point 2 is the experimental value; curve 3 is the value for film cooling in the pack with account for the droplets above the pack; 4 is the value for film cooling in the pack without account for the droplets above the pack. $T_{w0} = 29.4$ °C, $T_{a0} = 25.8$ °C, $\psi = 41.8\%$, specific flow rate $Q_w = 1.48$ kg/m^2 s.

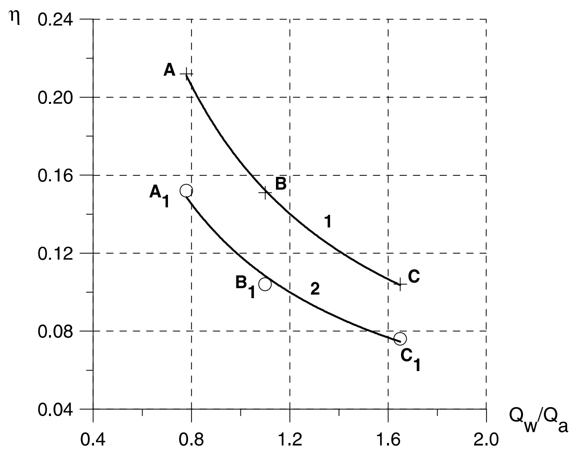

Fig. 6. Thermal efficiency of the cooling tower η as a function of the ratio between the mass flow rates of water and air Q_w/Q_a. Curve 1 is for the cooling tower as a whole; curve 2 is the contribution of film cooling in the pack. Points A and A_1 correspond to $T_{w0} = 34.4$ °C; B and B_1—29.4 °C; C and C_1—26.9 °C.

increases. The main reason is the enhancement of the rate of air flow through the cooling tower.

For the same hydraulic load and different initial temperatures of water, but under rather similar atmospheric conditions, the experimental data and calculated results on the change in the thermal efficiency of the cooling tower are displayed in Fig. 7. From the point of view of evaporation kinetics, the initial water temperature and the parameters of the inlet air are described by means of the parameter S. The parameter S is defined as

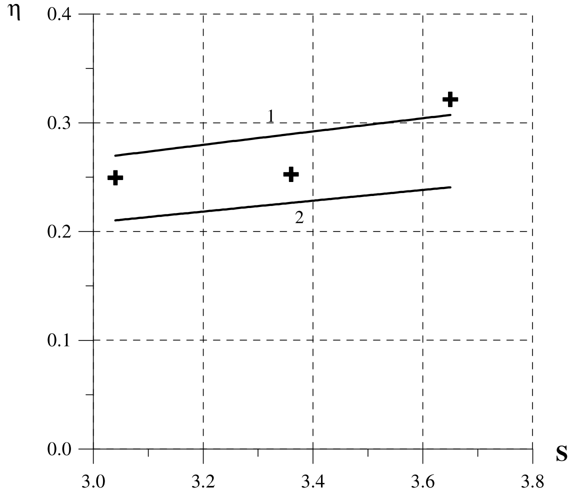

Fig. 7. Thermal efficiency of the cooling tower η versus the parameter S. Curve 1 is for the cooling tower as a whole; curve 2 is the contribution of film cooling in the pack; +, experimental values. $Q_w = 0.8$ kg/m^2 s.

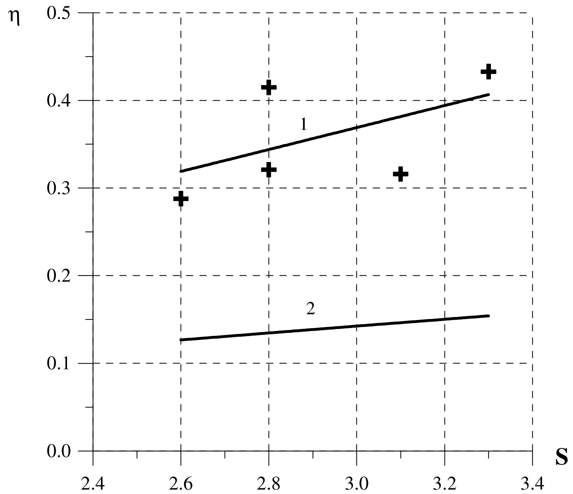

Fig. 8. Thermal efficiency of the cooling tower η versus the parameter S. Curve 1 is for the cooling tower as a whole; curve 2 is the contribution of film cooling in the pack; +, are experimental values. $Q_w = 2$ kg/m^2 s.

$$S = \frac{\rho_s(T_0)}{\rho_s(T_a)\psi}.$$

For a greater hydraulic load of a cooling tower, the same dependence is shown in Fig. 8. Curves 2 in these figures show the water film cooling in a cooling tower (with account for the influence of heat and mass transfer on droplets). We note that at a small hydraulic load (Fig. 7) the contribution of evaporative cooling of water films is more than 80% of the overall thermal efficiency of a cooling tower. One of the experimental values of the cooling tower efficiency is used to find the effective radius by the above-described technique. Further, for the given set of the experimental data this radius, corrected with account for the change in the surface tension with the initial water temperature [20], was used. We recall that for the analysis of these experimental data the coefficient k was determined beforehand. It is seen in Fig. 8, that at a large enough hydraulic load, when the effective radius of droplets is rather small, the contribution of evaporative cooling of droplets is not less than the contribution of film cooling. The contribution of the evaporative cooling of water films in the overall thermal efficiency of a cooling tower is at least more than 50%. Moreover, the decrease in the droplet radius on increase in the water temperature (the effect of temperature dependence of the surface tension of water) explains the steeper growth of the thermal efficiency of a cooling tower with S, which for these conditions is practically equivalent to an increase in the water temperature. It is obvious that the parameter S does not provide a sufficiently accurate description of evaporative cooling of water and should be used as an additional parameter to the basic one: the ratio of the mass flow rates of water and air.

According to Eq. (21), for relatively large S the rate of change in the temperature of droplets is directly proportional to the density of saturated water vapor, and, as is known, the density of saturated vapor is the exponential function of temperature. This fact can be used in the analysis of experimental data.

The flow rate of the evaporated water is an important parameter of the cooling tower performance in a steady-state regime. The model makes it possible to calculate the increase in the temperature and density of water vapor in air during its passage through the heat and mass transfer zones of the cooling tower. It was found that the evaporated water flow rate ranges from 0.5% to 1% of the circulating water flow rate in the cooling tower. This value depends on the difference between the densities of water vapor $\rho_s(T_{\text{in}})(1-1/S)$, on the velocity of the moist air flow in the cooling tower pack and on the atmospheric pressure.

4. Discussions of results

In this work, we present a mathematical model of evaporative cooling of water in a natural draft cooling tower. The model describes self-consistent evaporative cooling of falling droplets and water films on vertical sheets of a pack jointly affecting the velocity and other parameters of the ascending flow of moist air in the cooling tower. Aerodynamic processes in the cooling tower are described in one-dimensional approximation. It is shown that the simulation results qualitatively well describe all the experimental regularities; if one fitting parameter is used (effective droplet radius), there is a remarkably good quantitative agreement between our simulation results and experimental data.

Now we consider the applicability of one-dimensional model of evaporative cooling of water films, in other words, the use of only average temperatures of the elements of film and air flow. For example, a more accurate equation for describing the air temperature between the sheets of the pack can be written as follows:

$$v_a \partial_z T_a(z,y) = a\partial_{yy} T(z,y),$$

where y is the transverse coordinate. We will make qualitative evaluation. The characteristic time of hear transfer in the transverse direction $\tau_\perp \sim d^2/a$, and the characteristic time of heat transfer in the longitudinal direction is $\tau\| \sim H_f/v_a$. For effective use of the one-dimensional approximation it is necessary that inequality $\tau_\perp \ll \tau_\|$ be valid. Or

$$\frac{d^2 v_a}{aH_f} \ll 1. \tag{35}$$

The Galerkin method allows one to obtain a stronger inequality [25]:

$$\frac{d^2 v_a}{aH_f\pi^2} \ll 1. \tag{36}$$

To investigate the water vapor diffusion, it is natural to use the coefficient of diffusion D instead of the thermal diffusivity. In relation to the water film flow, the distance between the sheets of the pack d should be replaced by the film thickness h and the corresponding average velocity of water in the film v_f should be used.

Let us make numerical estimations using the formulas obtained above. For a laminar water film of thickness $h \sim 10^{-3}$ m and velocity $v_f \sim 0.02$ m/s and $H_f \sim 1$ m, inequality (36) is valid, and the use of our one-dimensional approach is justifiable. For a moist air between the sheets of the pack of a cooling tower, where $d \sim 2\times10^{-2}$ m, and $v_a \sim 1$ m/s the situation is much more complicated. Inequality (36) is valid for heat transfer processes in a laminar flow. At the same time, inequality (36) is not valid for diffusion transfer of water vapor in a laminar mode of flow. The Reynolds number for such a flow between the sheets is about 10^3, i.e., we have a transition regime. In other words, the two-dimensional consideration of the diffusion process is appropriate and makes it possible to refine our results. This problem is the subject of our next paper.

We note that the developed mathematical model of evaporative cooling of water in a natural draft cooling tower can be used for creating the system controlling a cooling tower, which is an important part of a power plant. The system controlling the cooling tower can significantly increase its economical efficiency in the case

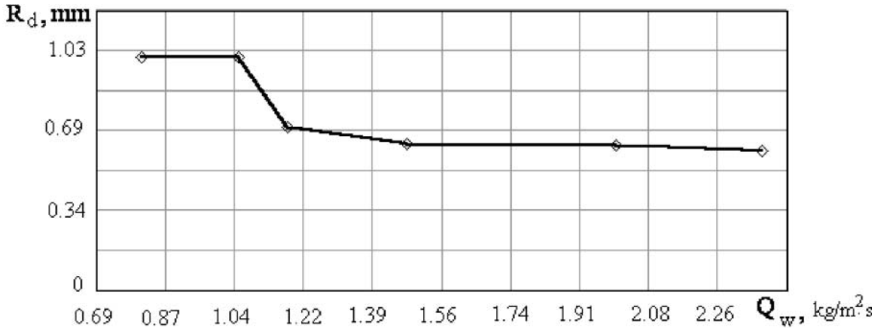

Fig. 9. Restored effective droplet radius as a function of hydraulic load.

of a variable hydraulic load, wind velocity, temperature and humidity of the inlet air [13].

Our mathematical model of evaporative cooling makes it possible to determine the value of some internal parameters in a natural draft cooling tower, which are difficult to measure directly, on the basis of the data of heat measurements of the inlet and outlet water and environmental atmosphere. The value of an effective droplet radius is shown in Fig. 9 as a function of the hydraulic load of a cooling tower. These data were obtained in processing our experimental data by means of our mathematical model and iterative algorithm. It can be seen that for small hydraulic load, the effective droplet radius is of about 1mm, and becomes almost half as large for a nominal hydraulic load. As noted above, the efficiency of evaporative cooling of droplets increases here almost three times. This effect is also clear by visible in Fig. 8.

For simulation of the cooling tower performance, there are a number of unsolved problems. We may note such urgent problems as the simulation of the cooling tower performance in winter conditions (Russia, Commonwealth of Independent States, and Scandinavia, etc.), when the aerodynamics of a cooling towers changes drastically and the simulation of the cooling tower performance under strong winds.

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